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A small non-vanishing cosmological constant from vacuum energy:physically and observationally desirable

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Increasing improvements in the independent determinations of the Hubble constant and the age of the universe now seem to indicate that we need a small non-vanishing cosmological constant to make the two independent observations consistent with each other. The cosmological constant can be physically interpreted as due to the vacuum energy of quantized fields. To make the cosmological observations consistent with each other we would need a vacuum energy density, $\rho_v \sim (10^{-3} eV)^4$ today (in the cosmological units $\hbar = c = k = 1$). It is argued in this article that such a vacuum energy density is natural in the context of phase transitions linked to massive neutrinos. In fact, the neutrino masses required to provide the right vacuum energy scale to remove the age Vs Hubble constant discrepancy are consistent with those required to solve the solar neutrino problem by the MSW mechanism.

1 Introduction

Increasing accuracy in astronomical observations is leading us to an increasing precision in the determination of cosmological parameters. This in turn is leading us to critically re-examine our cosmological models. In particular, the precise determination of the Hubble constant and the independent determination of the age of the universe is forcing us to critically re-examine the simplest and most appealing cosmological model - a flat universe with a zero cosmological constant[1, 2].

The Hubble constant enters in the relationship between the recession velocity of an object and it's distance from us. The recession velocity of an object can be determined by using the Doppler effect and is relatively easy to determine. It is the calibration of the extragalactic distance ladder which is the difficult part of measuring the Hubble constant and in which the precision has been increasing significantly. It is the use of the Cepheid variables as standard candles that has allowed the improved determination of the extragalactic distance scale. Cepheids are variable stars whose pulsation period are very stongly correlated with their luminosities. These stars are well understood theoretically and the period - luminosity relationship is well-documented empirically. By observationally determining the pulsation period of a Cepheid variable and using the period luminosity relationship one can immediately determine the luminosity of the object. Then by using the apparent brightness of the object one can accurately determine the distance to the Cepheid. Pierce et al[1] have used this technique with ground-based observations to determine the extragalactic distance scale. An excellent discussion on this subject is contained in the article by Pierce et al[1] and references therein.

Pierce et al[1] have determined the Hubble constant to be, $H_o = 87 \pm 7 km\ s^{-1}\ Mpc^{-1}$. They further point out in their article that this value of the Hubble constant is in fact in conflict with the independent determination of the age of the universe[3] using Galactic

globular clusters if we use the standard cosmological model with a zero cosmological constant. The estimate of the age derived from an analysis of the galactic globular clusters is $16.5 \pm 2\ Gyr$.

An accurate determination of the Hubble constant is also one of the important goals of the Hubble Space Telescope (HST). In fact, Freedman et al [2] have used the HST to calibrate the extragalactic distance scale and hence determine the Hubble constant. They obtainn the value of the Hubble constant to be $80 \pm 17 km\ s^{-1}\ Mpc^{-1}$. They also point out that their determination of the Hubble constant is inconsistent with the age of the globular clusters within the framework of standard $\Omega = 1$ cosmology with no cosmological constant.

2 Resolution of the age Vs. Hubble constant problem through the introduction of a small vacuum energy.

One of the ways to avoid the apparent conflict between the observed age of the universe and the observed Hubble constant is to introduce a small cosmological constant in the Einstein equations that govern the evolution of the universe. This idea has been extensively studied by a number of people including Tayler[5] and Klapdor and Grotz[6].

Let's quickly summarize how a cosmological constant of the right magnitude can solve the apprent conflict between the age and the Hubble constant. This can be seen from the following analysis[4]. To a good approximation our universe is spatially homogenous and isotropic on large scales. It is therefore appropriate to describe space-time by Robertson-Walker metric which can be written in the form, (units $c = 1$)

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{1}$$

where, (t, r, θ, ϕ) are the comoving coordinates describing a space time point and

$R(t)$ is the cosmic scale factor. Also, $k = +1, -1$ or 0 depending on when the universe is closed, open or flat.

The present expansion age of a matter dominated universe can be evaluated in a Robertson-Walker universe. General fine-tuning arguments as well as the inflationary picture gives us a preference for a flat universe, $\Omega_o = 1$. In this case, $t_o = \frac{2}{3}H_o^{-1}$. For, $\Omega_o \simeq 1$, one can expand the above expressions in a Taylor expansion,

$$t_o = \frac{2}{3}H_o^{-1}\left[1 - \frac{1}{5}(\Omega_o - 1) + ...\right] \tag{2}$$

We can also determine the present age of the universe containing both matter and vacuum energy such that $\Omega_{vac} + \Omega_{matter} = 1$,

$$t_o = \frac{2}{3}H_o^{-1}\Omega_{vac}^{-1/2}\ln\left[\frac{1 + \Omega_{vac}^{1/2}}{(1 - \Omega_{vac})^{1/2}}\right] \tag{3}$$

This will give us much longer lifetimes as can be seen most dramatically by examining the limit $\Omega_{vac} \rightarrow 1$ in which case $t_o \rightarrow \infty$. Indeed having an $\Omega_{vac} \sim 0.8$ is one of the possible solutions of the age Vs. Hubble constant discrepancy as can be seen from the following discussion.

First, let us quickly recall the observational numbers on the Hubble constant and the ages of globular clusters. Here it is worthwile pointing out that there are actually 2 sets of numbers which though consistent with each other have slightly different central values and error estimates. Pierce et al[1] quote the result of their analysis as yielding a Hubble constant, H_o of $87 \pm 7 km\ s^{-1}\ Mpc^{-1}$. They then draw attention to the fact that this is in conflict with the age estimate of the globular clusters inferred by VandenBergh's[3] analysis which gives an age of $16.5 \pm 2\ Gyr$.

Freedman et al[2] have a greater amount of data that they have analyzed very thoroughly. Thus they found over 20 Cepheids in a Virgo cluster galaxy as opposed to 3 Cepheids found by Pierce et al. They obtain a slightly different central value of H_o with

substantially larger and perhaps more realistic error bars than Pierce et al. They thus quote a Hubble constant value, $H_o = 80 \pm 17 km\ s^{-1}\ Mpc^{-1}$. Also based on a wider spectrum of data for ages they quote a central value of the age of the universe as 14 Gyr with 1 σ error bars of ± 2 Gyr. Freedman et al, however, too point out that even with their more generous error bars there is still a discrepancy between the Hubble constant and the age of the universe if we restrict ourselves to a standard $\Omega = 1$ cosmology with a zero cosmological constant.

Let us now see how a non-zero cosmological constant can solve this discrepancy and what values of the cosmological constant are typically required to minimally solve the discrepancy between the Hubble constant and the age of the universe. First, let us consider the lowest possible value of the Hubble constant as quoted by Freedman et al which is 63 $km\ s^{-1}\ Mpc^{-1}$. Let us consider this together with the age of the universe, $t_o = 14.5\ Gyr$ which though slightly higher than the central value quoted by Freedman et al is at the lower limit of the age quoted by Pierce et al. These two values of H_o and t_o imply a value of $H_o\ t_o = 0.93$. This corresponds to the value of $\Omega_{vac} = 0.66$. Thus, $\Omega_{vac} = 0.66$ would remove the contradiction between the Hubble constant as determined by Freedman et al and the age of globular clusters. However, let us consider a few more values of Ω_{vac}, H_o and t_o to see where future more precise observations and analysis might lead us. Consider $\Omega_{vac} = 0.7$ which implies $H_o\ t_o = 0.964$. For $H_o = 63 km\ s^{-1}\ Mpc^{-1}$ this would imply the age $t_o = 15\ Gyr$ and for $t_o = 14.5\ Gyr$ this would imply a $H_o = 65 km\ s^{-1}\ Mpc^{-1}$. Similarly, $\Omega_{vac} = 0.8$ would imply $H_o\ t_o = 1.076$ which would give an $H_o = 73 km\ s^{-1}\ Mpc^{-1}$ for $t_o = 14.5\ Gyr$ and for a $t_o = 16.5\ Gyr$ would imply $H_o = 64 km\ s^{-1}\ Mpc^{-1}$. Finally, if we consider the lower bound implied by the numbers quoted by Pierce et al, viz. $H_o = 80 km\ s^{-1}\ Mpc^{-1}$ and $t_o = 14.5\ Gyr$, we get $\Omega_{vac} \sim 0.85$. Clearly however one would not like to push the values of Ω_{vac} much higher than this number for a number of reasons. First, we would like $\Omega_{vac} + \Omega_{matter} = 1$ and

higher values of Ω_{vac} will start to conflict with the lower bound on matter density from galaxies and clusters. Furthermore, one can start to place an independent constraint on the cosmological constant fromm gravitational lens statistics. Thus based on the HST Snapshot survey of quasars Maoz and Rix[35] have inferred the bound $\Omega_{vac} \leq 0.7$. Fukugita, Hogan and Peebles[36] have stated that this constraint may be relaxed slightly if one considers the fact that at large distances typical of lensing events galaxies are observed to have larger star formation rates than their nearby counterparts and may also contain larger quantities of dust. This may lead to some lensing events getting obscured and hence the upper limit for Ω_{vac} may rise if this is taken into account. It is unclear how much the upper limit will be pushed upwards but the fact remains that for both the above reasons one would not like an extremely large value of Ω_{vac} even though a value of $\Omega_{vac} \sim 0.8$ perhaps best meets all the observational constraints outlined above. Improved more accurate results and more exhaustive analysis on all these fronts will clearly shed further light on all these connected issues. Let us now turn our attention to a discussion of the possible physical origin of such an Ω_{vac}.

The value $\Omega_{vac} \sim 0.8$ corresponds to an energy density of the vacuum energy density of $\rho_v \sim (10^{-47} GeV^4)$. This energy density is of course much lower than most familiar energy scales in particle physics and the question naturally arises as to the physical origin of this energy scale. The smallness of this energy scale has been frequently referred to as the cosmological constant problem.

However, we'll argue in this paper that in fact a cosmological constant of the right magnitude required to make the cosmological observations consistent with each other may follow from the dynamical evolution of our universe. The basic physical picture which will allow us to arrive to this conclusion is that the cosmological constant might be interpreted as the vacuum energy of the quantized fields.

This point has been made by many people and is discussed at length by Birell and

Davies[7]. Further, this vacuum energy is not a static quantity but a function of time. This idea too has been extensively explored by a number of people including Peebles and Ratra[8], Freese, Adams, Frieman and Mottola[9], Reuter and Wetterich[10]. In fact, we know that there were a number of phase transitions in the evolution of the universe.

Thus, the history of the universe may be summarized as periods of dramatic change charecterized usually by phase transitions with relatively quiet periods of relaxation between the phase transitions [4]. Indeed, since the vacuum energy density changes as the $(characteristic\ \ energy\ \ scale)^4$ at a phase transition, in the absence of fine-tuning one expects that the vacuum energy density at the end of a phase transition $\sim (characteristic\ \ energy\ \ scale)^4$

This idea has been spelt in detail in the paper by Wilczek[11] and also by Reuter and Weterich[10]. Thus, $\rho_v \sim (10^{15}\ GeV)^4$ at the Grand Unified Symmetry breaking, $\rho_v \sim (10^2\ GeV)^4$ at the elctroweak symmetry breaking and $\rho_v \sim (10^{-1}\ GeV)^4$ at the chiral symmetry breaking in QCD.

Furthermore, at the conclusion of a phase transition the vacuum energy starts decaying more slowly to the energy scale characterized by next phase transition. This point of view is implicit in the papers by Wilczek[11] and has been explicitly stated by Reuter and Wetterich. In fact, the physical mechanism for the decay of vacuum energy is coupling to lighter fields. This mechanism is briefly discussed by Freese, Adams, Frieman and Mottola[9], who do an extensive analysis of cosmology with a decaying vacuum energy.

Thus, the question of the magnitude of the cosmological constant really becomes a question about energy scales. Almost every paper on the subject of the cosmological constant has had to struggle with the characteristic energy scale of $\sim 10^{-3} eV$ in the form $\rho_v \sim (0.003 eV)^4$ in various guises such as $(10^{-47} GeV^4)$ or $\Lambda/M_{pl}^2 \sim 10^{-120}$

The fact that $0.003eV$ is so much less than any characteristic energy which familiar to most of us has caused a great deal of consternation.

However, the energy scale 0. 003eV is certainly not a complete stranger to us. The most natural low-energy scale that particle physics gives us is the light neutrino masses that follow from the see-saw model of neutrino masses. In fact, the neutrino masses required to to solve the solar neutrino problem by MSW mechanism[12] imply neutrino masses $\sim 10^{-3} eV$

This, of course, is a powerful hint but is not yet a solution to the cosmological constant problem.

In fact, the finite temperature behaviour of the see-saw model of neutrino masses has been studied in detail by Holman and Singh[26]. The original motivation for studying this model was to provide a concrete particle physics model fo the Late Time Phae Transitions model for structure formation. Our analysis showed that in fact this model does exhibit a phase transition with a critical temperature $T_c \sim (few)\ \ m_\nu$.

3 Late Time Phase Transitions and the time evolution of the Hubble parameter

In this section we will discuss the cosmological motivations and particle physics models for Late Time Phase Transitions. Once we have a specific model we will study the time evolution of fields and the scale factor in this model. In particular, we will be interested in studying the time evolution of the Hubble parameter and will see that in this model the Hubble " constant ", in fact has an acceptable value at the present age of the universe.

Phase transitions that occur after the decoupling of matter and radiation have been discussed in the literature as Late Time Phase Transitions (LTPT's). The original motivation for considering LTPTs[14] [19] [20] [22] [23] [26]was the need to reconcile the extreme isotropy of the Cosmic Microwave Background Radiation (CMBR)[15] with the existence of large scale structure[16] and also the existence of quasars at high redshifts[17].

Discussions of realistic particle physics models capable of generating LTPT's have

been carried out by several authors[23] [22]. It has been pointed out that the most natural class of models in which to realise the idea of LTPT's are models of neutrino masses with Pseudo Nambu Goldstone Bosons (PNGB's). The reason for this is that the mass scales associated with such models can be related to the neutrino masses, while any tuning that needs to be done is protected from radiative corrections by the symmetry that gave rise to the Nambu-Goldstone modes[24].

Holman and Singh[26] studied the finite temperature behaviour of the see-saw model of neutrino masses and found phase transitions in this model which result in the formation of topological defects. In fact, the critical temperature in this model is naturally linked to the neutrino masses.

The original motivation for studying the finite temperature behaviour of the see-saw model of neutrino masses came from a desire to find realistic particle physics models for Late Time Phase Transitions. It now appears that this may also provide a physically appealing and observationally desirable magnitude for the cosmological constant.

In particle physics one of the standard ways of generating neutrino masses has been the see-saw mechanism [25]. These models involve leptons and Higgs fields interacting by a Yukawa type interaction. We computed the finite temperature effective potential of the Higgs fields in this model. An examination of the manifold of degenerate vacua at different temperatures allowed us to describe the phase transition and the nature of the topological defects formed.

To investigate in detail the finite temperature behaviour of the see-saw model we selected a very specific and extremely simplified version of the general see-saw model. However, we expect some of the qualitative features displayed by our specific simplified model to be at least as rich as those present in more complicated models.

3.1 A particle physics model for LTPT

We chose to study the 2-family neutrino model. Because of the mass hierarchy and small neutrino mixings [12] we hope to capture some of the essential physics of the ν_e-ν_μ system in this way. The 2-family see-saw model we consider requires 2 right handed neutrinos N_R^i which transform as the fundamental of a global $SU_R(2)$ symmetry. This symmetry is implemented in the right handed Majorana mass term by the introduction of a Higgs field σ_{ij}, transforming as a symmetric rank 2 tensor under $SU_R(2)$ (both N_R^i and σ_{ij} are singlets under the standard model gauge group). The spontaneous breaking of $SU_R(2)$ via the vacuum expectation value (VEV) of σ gives rise to the large right handed Majorana masses required for the see-saw mechanism to work. Also, the spontaneous breaking of $SU_R(2)$ to $U(1)$ gives rise to 2 Nambu Goldstone Bosons. The $SU_R(2)$ symmetry is explicitly broken in the Dirac sector of the neutrino mass matrix, since the standard lepton doublets l_L and the Higgs doublet Φ are singlets under $SU_R(2)$. It is this explicit breaking that gives rise to the potential for the Nambu Goldstone modes via radiative corrections due to fermion loops. Thus, these modes become Pseudo Nambu Goldstone Bosons (PNGB's).

The relevant Yukawa couplings in the leptonic sector are:

$$- \mathcal{L}_{\rm yuk} = y_{ai} \bar{l_L}^a N_R^i \Phi + y \overline{N_R^i} N_R^{j\ c} \sigma_{ij} + \text{h.c.} \tag{4}$$

where $a, i, j = 1, 2$. The $SU_R(2)$ symmetry is implemented as follows:

$$\begin{aligned} N_R^i &\rightarrow U^i_j \; N_R^j \\ \sigma_{ij} &\rightarrow U_i^k \sigma_{kl} (U^T)^l_j \end{aligned} \tag{5}$$

where U^i_j is an $SU_R(2)$ matrix. The first (Dirac) term above transforms as an $SU_R(2)$ doublet, thus breaking the symmetry explicitly.

We now choose the VEV of σ to take the form[27]: $\langle\sigma_{ij}\rangle = f\delta_{ij}$, thus breaking $SU_R(2)$ spontaneously down to the $U(1)$ generated by τ_2 (where τ_i are the Pauli matrices). We take f to be much larger than the doublet expectation value v.

We can parametrize σ_{ij} so as to exhibit the Nambu-Goldstone modes as follows:

$$\begin{aligned}\sigma(x) &= U(x)\langle\sigma\rangle U^T(x) \\ &= fU(x)U^T(x)\end{aligned} \tag{6}$$

with $U(x) = \exp(i(\xi_1\tau_1 + \xi_3\tau_3)/f)$ (note that U is symmetric).

After the Higgs doublet acquires its VEV, we have the following mass terms for the neutrino fields:

$$-\mathcal{L}_{\text{mass}} = m_{ai}\bar{\nu}_L^a N_R^i + M\overline{N}_R UU^T N_R^c + \text{h.c.} \tag{7}$$

where ν_L^a are the standard neutrinos, $m_{ai} = y_{ai}\ v/\sqrt{2}$, $M = yf/\sqrt{2}$.

Diagonalizing the neutrino mass matrix in the standard see-saw approximation ($|m_{ai}| << M$) and performing a chiral rotation to eliminate the γ_5 terms, we find that the ξ_i dependent *light* neutrino masses are given by

$$\begin{aligned}m_1^2 &= \frac{1}{M^2}[\cos^2 2||\xi||\ (m_{11}^2 + m_{12}^2)^2 + \sin^2 2||\xi||\ (\widehat{\xi}_3(m_{11}^2 - m_{12}^2) + 2m_{11}m_{12}\widehat{\xi}_1)^2] \\ m_2^2 &= \frac{1}{M^2}[\cos^2 2||\xi||\ (m_{21}^2 + m_{22}^2)^2 + \sin^2 2||\xi||\ (\widehat{\xi}_3(m_{21}^2 - m_{22}^2) + 2m_{21}m_{22}\widehat{\xi}_1)^2]\end{aligned} \tag{8}$$

where $||\xi|| = \sqrt{\xi_1^2 + \xi_3^2}/f$ and $\widehat{\xi}_i = \xi_i/\sqrt{\xi_1^2 + \xi_3^2}$. We neglect the effects of the heavier neutrinos since they will be suppressed by powers of m_{ai}/M in the loops that will generate the effective potential for the ξ_i's.

For simplicity, to begin with we shall restrict ourselves to the case where the Dirac mass matrix m_{ai} is proportional to the identity: $m_{ai} = m\ \delta_{ai}$. We will consider a more general case later. Using standard results on the computation of the effective potential

due to fermion loops[28] (we treat the ξ_i's as classical background fields, i. e. we do not allow them to propagate in loops), we can calculate the one-loop effective potential for the ξ_i's. The renormalized potential can be expressed as follows:

$$V_{\rm renorm}(\xi_1,\xi_3) = V_0 + m_r^2\mathcal{M}^2 + \lambda_r(\mathcal{M}^2)^2 - \frac{1}{8\pi^2}(\mathcal{M}^2)^2(\ln\frac{\mathcal{M}^2}{\mu^2} - \frac{1}{2}) \tag{9}$$

where μ is the subtraction point. Note that we can absorb the effects of λ_r by redefining μ. We will suppose that this has been done in what follows. Further, the quantity $\mathcal{M}^2$, is given by

$$\mathcal{M}^2 = \frac{m^4}{M^2}(\cos^2 2||\xi|| + \hat{\xi}_3^2\sin^2 2||\xi||). \tag{10}$$

The finite temperature correction due to the two light neutrinos is given by,

$$\Delta V_T(\xi_1,\xi_3) = -4\frac{T^4}{\pi^2}\int_0^\infty dx x^2 \log\left[1 + \exp - \left[x^2 + \frac{\mathcal{M}^2}{T^2}\right]^{1/2}\right] \tag{11}$$

The above expression can be evaluated numerically for any given choice of parameters, however for some purposes it is useful to expand the above expression to get analytic expressions. Performing the high temperature expansion of the complete potential and discarding terms of order $(\mathcal{M}^2)^3/T^2$ or higher we get,

$$V_{\rm tot}(\xi_1,\xi_3) = V(\mathcal{M}^2) = (V_0 - \frac{7\pi^2T^4}{90}) + (m_r^2 + T^2/6)\mathcal{M}^2 + \frac{(\mathcal{M}^2)^2}{8\pi^2}(n - \log\frac{T^2}{\mu^2}), \tag{12}$$

where $n = 2\gamma - 1 - 2\log\pi \sim -2.1303$, m_r is a parameter in the model and μ is the renormalisation scale. $\mathcal{M}$ is naturally of the neutrino mass scale in this model.

A study of the manifold of degenerate vacua of the effective potential at different temperatures revealed phase transitions in this model accompanied by the formation of topological defects at a temperature of a few times the relevant neutrino mass . Typically at higher temperatures the manifold of degenerate vacua consisted of a set of disconnected points whereas at lower temperatures the manifold was a set of connected circles. Thus,

domain walls would form at higher temperatures which would evolve into cosmic strings at lower temperatures.

Since the critical temperature of the phase transition, $T_c \sim (few)\ m_\nu$, let us quickly summarise the observational evidence for a small m_ν.

At neutrino detectors around the world, fewer electron neutrinos are received from the sun than predicted by the Standard Solar Model. An explanation of the deficiency is offered by the MSW mechanism[12] which allows the ν_e produced in solar nuclear reactions to change into ν_μ. This phenomenon of neutrino mixing requires massive neutrinos with the masses for the different generations different from each other [12].

The model we considered earlier was an extremely simple one. Although it had 2 families of light neutrinos, there was only one single light neutrino mass. As such this model was not compatible with the MSW effect. However it is fairly straightforward to modify our original model to make it compatible with the MSW effect as is shown in what follows.

To ensure that it is not possible to choose the weak interaction eigenstates to coincide with the mass eigenstates we must require the 2 neutrino mass scales to be different. We can ensure neutrino mixing in our model by demanding that m_{ai} be such that $m_{11} \neq m_{22}$ and $m_{12} = 0 = m_{21}$. In this case, the effective potential $V_{\text{tot}}(\xi_1, \xi_3) = 1/2(V(\mathcal{M}_1^2) + V(\mathcal{M}_2^2))$ with $V(\mathcal{M}_i^2)$ having the same functional form as $V(\mathcal{M}^2)(i = 1, 2)$ and $\mathcal{M}_i^2$ given by the following expression:

$$\mathcal{M}_i{}^2 = \frac{m_{ii}^4}{M^2}(\cos^2 2||\xi|| + \hat{\xi}_3^2 \sin^2 2||\xi||) \tag{13}$$

.

Further, if $m_{11} << m_{22}$ then $V_{\text{tot}}(\xi_1, \xi_3) = V(\mathcal{M}_2^2)/2$, which is exactly half the finite temperature effective potential we discussed earlier except the neutrino mass scale is the heavier neutrino mass scale. Hence, the discussion on phase transitions and formation

of topological defects we carried out earlier goes through exactly except that the critical temperature is determined by the mass scale of the heavier of the 2 neutrinos.

In the complete picture of neutrino masses[12], the neutrinos might have a mass hierarchy analogous to those of other fermions. Further, we expect that the mixing between the first and third generation might be particularly small . In this scheme, it is a good first approximation to consider 2-family mixing. We are here particularly interested in the ν_e-ν_μ mixing. This is also the mixing to which the solar neutrino experiments are most sensitive. A complete exploration of MSW solutions to the solar neutrino problem has recently been reported by Shi, Schramm and Bahcall[29]. We shall restrict ourselves to the 2-family mixing. The data seems to imply a central value for the mass of the muon neutrino to be a few meV[30].

We now turn to a quick discussion of the distortions of the Cosmic Microwave Background Radiation (CMBR) this model produces. The most significant microwave distortion comes from collapsing domain wall bubbles. This has been discussed and calculated by Turner, Watkins and Widrow (TWW)[18]. As pointed out by TWW this anisotropy is most significant on $\sim 1^o$ angular scales. The temperature shift due to a photon traversing a collapsing domain wall bubble is

$$\frac{\Delta T}{T} = 2.64 \times 10^{-4} h^{-1} \beta A \sigma / (10 MeV^3) \tag{14}$$

where h, A, β are dimensionless numerical constants of order unity and σ is the surface tension of the domain wall. The present measurements of the CMBR anisotropy then imply[34] that $\sigma < 0.5 MeV^3$.

An estimate of σ in terms of the quantities m_ν and f introduced in our model can be obtained[13]. (To make contact with the work of L. Widrow cited above please note that his $\lambda m^4 = m(\nu_\mu)$ and $m = f$ in our notation.) Thus, the constraint on σ then implies that $f < 10^{15} GeV$. Our model is clearly an effective theory with f being some higher

symmetry breaking scale on which it is tough to get an experimental handle. However, the constraint derived above is in fact natural in the context of the see-saw model of neutrino masses embedded in Grand Unified Theories as discussed by Mohapatra and Parida (MP)[32] and also by Deshpande, Keith and Pal(DKP)[33].

3.2 Time evolution in the LTPT model

Now that we know the potential in which the fields ξ_1 and ξ_3 evolve, we can write down the coupled set of evolution equations which describe the time evolution of the fields and the scale factor of the universe. Once again we'll follow the general techniques described in Kolb and Turner[4]. The time evolution of the scale factor is given by equations like (2), (3) and (4). It is perhaps worth noting that the expression for the presssure and energy density of the fields is given by:

$$\rho_\xi = \frac{\dot{\xi}_1^2 + \dot{\xi}_3^2}{2} + V(\xi_1, \xi_3) \tag{15}$$

$$p_\xi = \frac{\dot{\xi}_1^2 + \dot{\xi}_3^2}{2} - V(\xi_1, \xi_3) \tag{16}$$

The time evolution of the fields ξ_1 and ξ_3 in an FRW universe is given by,

$$\ddot{\xi}_1 + 3\frac{\dot{R}(t)}{R(t)}\dot{\xi}_1 + \frac{\partial V}{\partial \xi_1} = 0 \tag{17}$$

$$\ddot{\xi}_3 + 3\frac{\dot{R}(t)}{R(t)}\dot{\xi}_3 + \frac{\partial V}{\partial \xi_3} = 0 \tag{18}$$

These coupled equations describing the time evolution can be solved numerically. Here we are interested in the time evolution at very recent epochs. Clearly, for extremely recent epochs the high temperature expansion is inappropriate. However, the zero temperature potential which we have computed and described is a good approximation for stuudying the time evolution at recent epochs. Thus we will use the zero temperature potential to do the time evolution in what follows. In fact as it turns out the time evolution of the

scale factor is fairly insensitive to the initial conditions on the fields but is determined primarily by the order of magnitude of the energy density in the fields. In fact we evolved the system with a variety of initial conditions on the fields and observed an almost identical time evolution for the scale factor.

We have investigated the behaviour of the system for a variety of choice of parameters entering into the potential with the typical order of magnitude for the parameters $m_r \sim m_\nu$ and $V_o \sim m_\nu^4$. Since it is the fact that the vacuum energy density in our model is $\sim m_\nu^4$ that plays a crucial role in the time evolution in our model, let us re-emphasise why this is natural in the context of our model.

Recall, that the original $-\mathcal{L}_{\rm yuk}$ contained 2 distinct couplings, y_{ai} and y. In particular, if we set $y_{ai} = 0$ in the Lagrangian then the symmetry of the Lagrangian was enhanced, the light neutrino masses would have been identically zero and the potential for the ξ_i fields would have also been identically zero. If $y_{ai} \neq 0$ then the $SU_R(2)$ symmetry is explicitly broken, the light neutrino fields pick up their masses as outlined earlier and also the ξ_i's develop a non-trivial potential. Furthermore, since setting $y_{ai} = 0$ enhances the symmetry of the Lagrangian, there is a symmetry which protects the small parameters in this model. In the model we are studying the coupling of the light neutrino fields to the ξ fields is identically zero if $y_{ai} = 0$ and arises at the second order in perturbation theory in the see-saw diagonalization if $y_{ai} \neq 0$. It is the coupling of the neutrino fields to the ξ fields that is responsible for a non-zero effective potential for the ξ fields, hence there is a prefactor to the entire effective potential (including the vacuum energy part) which is proportional to the appropriate power of y_{ai}. The observable quantity that y_{ai} corresponds to is m_ν as given in section 3.1 .Since the effective potential has four mass dimensions, the dimensional prefactor multiplying the potential ends up being m_ν^4. Thus it is natural that the contribution to the vacuum energy density due to the fields appearing in the see-saw model of neutrino masses presented here ends up being $\sim m_\nu^4$.

One, of course, has to worry about the contribution of other heavier fields to the vacuum energy density in the cosmological context. The underlying picture being used is that discussed by Wilczek[11], Reuter and Wetterich[10] and Freese, Adams, Frieman and Mottola[9]. They argue that the cosmological constant will decay during the evolution of the universe as the vacuum energy of heavier fields dissipates due to their coupling to lighter fields. One would then expect the vacuum energy density at late times to be dominated by the contribution of the lightest most weakly coupled fields such as those appearing in the see-saw model of neutrino masses discussed in this paper. An in-depth analysis of the details of this mechanism will be the subject of a later work.

The time evolution of the system can be summarized as follows. The fields evolved to the minimum of their potential on a time scale which is short compared to the typical Hubble time scale in the problem. The evolution of the scale factor follows the normal matter dominated behaviour for a while until the vacuum energy starts playing an important role. After this time the vacuum energy starts driving the time evolution of the scale factor. Thus it is the value of the vacuum energy density that determines the asymptotic time evolution of the system. For our model with the choice of parameters stated and rationalised above we have the final vacuum energy density $\simeq m_\nu^4$.

The observationally important plot is the plot of the Hubble parameter as function of time. This is displayed in figure 1. As you can see the Hubble parameter assumes a constant value after the vacuum energy starts playing the dominant role in the evolution of the scale factor.

In fact, in retrospect one can understand the time evolution of the coupled differential equations simply by noting the order of magnitude of the quantities involved in the evolution equations.

Let us introduce the following dimensionless physical quantities, $\tau = H\ t$, $\eta_i = \frac{\xi_i}{f}$ and $\mathcal{V} = \frac{V}{m_\nu^4}$ We'll use the following physical quantities to make the dimensionless quantities

of order 1, $H = 75\ h\ km\ sec^{-1}\ Mpc^{-1}$, $f = 10^{12}\ f_{12}\ GeV$ and $m_\nu = 2.5\ m_d \times 10^{-3} eV$.

Here are some of the quantities of interest and their magnitudes. First, the expressions for the pressure density and energy density of the fields is given below:

$$p_\eta = m_\nu^4 \left\{ 6.5 \times 10^{-14} \frac{h^2 f_{12}^2}{m_d^4} \left[\left(\frac{d\eta_1}{d\tau} \right)^2 + \left(\frac{d\eta_3}{d\tau} \right)^2 \right] - \mathcal{V}(\eta_1, \eta_3) \right\} \tag{19}$$

$$\rho_\eta = m_\nu^4 \left\{ 6.5 \times 10^{-14} \frac{h^2 f_{12}^2}{m_d^4} \left[\left(\frac{d\eta_1}{d\tau} \right)^2 + \left(\frac{d\eta_3}{d\tau} \right)^2 \right] + \mathcal{V}(\eta_1, \eta_3) \right\} \tag{20}$$

Since we have scaled quantities so that $m_d\ ,\ h,\ \eta_i,\ \tau\ and\ \mathcal{V}$ are all of order 1 it follows that,

$$\rho_\xi \simeq V(\xi_1, \xi_3) \tag{21}$$

$$p_\xi \simeq -V(\xi_1, \xi_3) \tag{22}$$

Thus in fact, for all practical purposes, we have

$$\rho_\xi \simeq -p_\xi \tag{23}$$

which is the equation of state for vacuum energy and hence demonstrates that this solution is very close in spirit to the cosmological constant solution for the age vs Hubble constant problem. What we have achieved is to provide a physical basis for the correct order of magnitude for this effective cosmological constant. This can be seen clearly by making the evolution equation for the scale factor dimensionless too.

$$\begin{aligned} &\frac{1}{R(\tau)} \frac{dR(\tau)}{d\tau} + \frac{k}{R^2(\tau) H^2} = \\ &\quad \Omega_m(\tau_0) \left(\frac{R(\tau_0)}{R(\tau)} \right)^3 + \Omega_r(\tau_0) \left(\frac{R(\tau_0)}{R(\tau)} \right)^4 \\ &\quad + \left\{ \frac{8\pi}{3} 4.0 \times 10^{-8} \frac{h^2 f_{12}^2}{m_d^4} \left[\left(\frac{d\eta_1}{d\tau} \right)^2 + \left(\frac{d\eta_3}{d\tau} \right)^2 \right] + 0.85 \frac{m_d^4}{h^2} \mathcal{V}(\eta_1, \eta_3) \right\} \end{aligned}$$

Note that it is the fact that the mass of the neutrinos is the correct order of magnitude which allowed the age of the universe to become compatible with the Hubble constant observed today.

The picture presented in this article is of a vacuum energy that changes as a function of time due to the coupling of the fields responsible for the vacuum energy to the other fields. One may worry therefore that the vacuum energy may disappear because of the coupling of the ξ fields to other fields. However because ξ can only decay into the lighter neutrinos it is coupled to, the time scale on which this vacuum energy will dacay is much larger than the present age of the universe. This can be seen by calculating the decay width of the ξ , Γ_ξ.

This decay width of the ξ particles arises because of the coupling of the ξ to the lighter fermions with coupling y is given by[4]

$$\Gamma_\xi = \frac{y^2 m_\xi}{8\pi} \tag{24}$$

where,

$$m_\xi^2 \simeq \frac{\partial^2 V}{\partial \xi^2} \tag{25}$$

Therefore,

$$m_\xi^2 \simeq \frac{m_\nu^4}{f^2} \tag{26}$$

Thus the timescale on which the energy in the ξ fields is converted into the energy of ν's is given by

$$\Gamma_\xi^{-1} \simeq \frac{8\pi}{y^2 m_\nu} \frac{f}{m_\nu} \tag{27}$$

which is much greater than the present age of the universe. Thus this vacuum energy is clearly not a short lived thing. As the expression above displays this is a consequence

both of the fact that we have light particles involved and that they are extremely weakly coupled to other particles.

In conclusion, the MSW solution to the solar neutrino problem seems to imply a muon neutrino mass of a few meV. This in turn would lead to a phase transition in the PNGB fields associated with massive neutrinos with a critical temperature of several meV. This would then give us a vacuum energy density $\sim (10^{-3}\ eV)^4$, which would help resolve the conflict between the independent determinations of the Hubble constant and the age of the universe. This phase transition also happens at the correct epoch in the evolution of the universe to provide a possible explanation of the peak in quasar space density at redshifts of 2 to 3[31].

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FIGURE CAPTIONS

Figure 1 : Time evolution of the Hubble Parameter in the LTPT model : $\frac{\dot{R}(t)}{R(t)}$ Vs. Time.